

# Traffic flow and efficient routing on scale-free networks: A survey

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Recently, motivated by the pioneer works in revealing the small-world effect and scale-free property of various real-life networks, many scientists devote themselves to studying complex networks. In this paper, we give a brief review on the studies of traffic flow and efficient routing on scale-free networks, including the traffic dynamics based on global routing protocol, Traffic dynamics based on local routing protocol, and the critical phenomena and scaling behaviors of real and artificial traffic. Finally, perspectives and some interesting problems are proposed.

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## I. INTRODUCTION

Many social, biological, and communication systems can be properly described as complex networks with vertices representing individuals or organizations and links mimicking the interactions among them [1, 2, 3, 4]. One of the ultimate goals of the current studies on topological structures of networks is to understand and explain the workings of systems built upon those networks, for instance, to understand how the topology of the World Wide Web affects Web surfing and search engines, how the structure of social networks affects the spread of diseases, information, rumors or other things, how the structure of a food web affects population dynamics, and so on. The increasing importance of large communication networks such as the Internet [5], upon which our society survives, calls for the need for high efficiency in handling and delivering information. Therefore, to understand the traffic dynamics and find the optimal strategies for traffic routing is one of the important issues we have to address. There have been many previous studies to understand and control traffic congestion on networks, with a basic assumption that the network has a homogeneous structure [6, 7, 8, 10, 46]. However, many real-life communication networks, such as the Internet [11] and the World-Wide-Web [12], display scale-free degree distribution [13, 14], thus it is of great interest to study the traffic flow on scale-free networks. In this light, the traffic dynamics on complex networks have recently attracted a large amount of interest from the physical community.

In this paper, we will give a brief review about traffic dynamics on scale-free networks. This paper is organized as follow: In Sec. 2 and 3, the traffic dynamics with global and local routing protocols are introduced, respectively. In Sec. 4, the critical phenomena and self-similarity scaling of real traffic and artificial models are discussed. Finally, we sum up this paper in Sec. 5.

## II. TRAFFIC DYNAMICS BASED ON GLOBAL ROUTING PROTOCOL

In this section, we discuss the case where the whole topological information is available for each router. For simplicity, all the nodes are treated as both hosts and routers. The simplest model can be described as follows:

(1) At each time step, there are  $R$  packets generated in the system, with randomly chosen sources and destinations. Once a packet is created, it is placed at the end of the queue if this node already has several packets waiting to be delivered to their destinations.

(2) At each time step, each node,  $i$ , can deliver at most  $C_i$  packets one step toward their destinations according to the routing strategy.

(3) A packet, once reaching its destination, is removed from the system.

We are most interested in the critical value  $R_c$  where a phase transition takes place from free flow to congested traffic. This critical value can best reflect the maximum capability of a system handling its traffic. In particular, for  $R < R_c$ , the numbers of created and delivered packets are balanced, leading to a steady free traffic flow. For  $R > R_c$ , traffic congestion occurs as the number of accumulated packets increases with time, simply for that the capacities of nodes for delivering packets are limited. To characterize the phase transition, we use the following order parameter

$$H(R) = \lim_{t \rightarrow \infty} \frac{C}{R} \frac{\langle \Delta W \rangle}{\Delta t}, \quad (1)$$

where  $\Delta W = W(t + \Delta t) - W(t)$ , with  $\langle \dots \rangle$  indicating average over time windows of width  $\Delta t$ , and  $W(t)$  is the total number of packets in the network at time  $t$ . Clearly,  $H$  equals zero in the free-flow state, and becomes positive when  $R$  exceeds  $R_c$ .

Since in the Internet, the deviation of traffic from the shortest path is only about 10% [15], one can assume that the routing process takes place according to the criterion of the shortest available path from a given source to its destination. Accordingly, firstly, we investigate the shortest-path routing strategy, which can be implemented by either of the two ways, finding the shortest-

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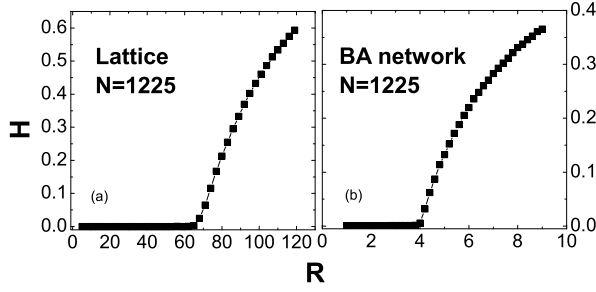


FIG. 1: The order parameter  $H$  versus  $R$  for two-dimensional lattice (a) and BA networks (b) with the same size  $N = 1225$  and average degree  $\langle k \rangle = 4$ . The delivering capability of each node is simply set as a constant  $C = 1$ . The shortest-path routing strategy yields  $R_c^{\text{Lattice}} \approx 60$  and  $R_c^{\text{BA}} \approx 4.0$ , respectively.

path dynamically or following the fixed routing table. In the former case [16], for each newly generated packet, the router will find a shortest path between its source and destination, and then, the packet is forwarded along this path during the following time steps. In the latter case [17], for any pair of source and destination, one of all the shortest paths between them is randomly chosen and put into the fixed routing table that is followed by all the information packets. Compared with the dynamical routing algorithm and the information feed-back mechanism, the fixed routing algorithm is much more widely used in real communication systems for its obvious advantages in economical and technical costs [18, 19]. Actually, the behaviors of those two cases are pretty much the same [16, 17], thus we will not distinguish them hereinafter.

If the delivering capability of each node is the same, the critical point  $R_c$  of highly heterogeneous network will be much smaller than that of homogeneous network. It is because that when all the packets follow their shortest paths, it will easily lead to the overload of the heavily-linked router, and the congestion will immediately spread over all the nodes. Fig. 1 shows the order parameter  $H$  versus  $R$  for (a) the two-dimensional lattice with periodical boundary condition and (b) the Barabási-Albert (BA) network [13, 14] with average degree  $\langle k \rangle = 4$ . Clearly, the throughput, measured by the  $R_c$ , of regular network is much larger than that of scale-free networks.

To provide the theoretical estimate of the value of  $R_c$ , we first introduce the concept of betweenness centrality (see also the original concept of centrality [20, 21], the generalized concept of centrality [22], the physical meaning of betweenness centrality [23], and some recently proposed algorithms for calculating betweenness [24, 25, 26]). The betweenness centrality of a node  $v$  is defined as

$$B_v = \sum_{s \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}, \quad (2)$$

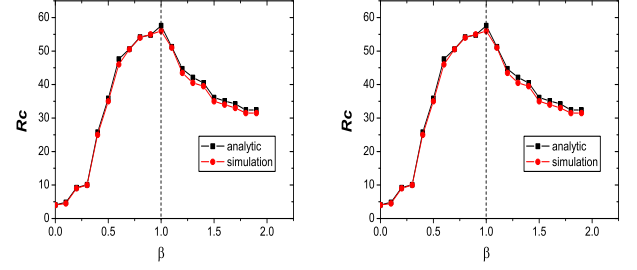


FIG. 2: (Color online) The critical  $R_c$  vs  $\beta$  for BA network with average degree  $\langle k \rangle = 4$  and of size  $N = 1225$  (left panel) and  $N = 1500$  (right panel). Both simulation and analysis demonstrate that the maximal  $R_c$  corresponds to  $\beta \approx 1$ . The results are the average over 10 independent runs.

where  $\sigma_{st}$  is the number of shortest paths going from  $s$  to  $t$  and  $\sigma_{st}(v)$  is the number of shortest paths going from  $s$  to  $t$  and passing through  $v$ . Below the critical value  $R_c$ , there is no accumulation at any node in the network and the number of packets that arrive at node  $v$  is, on average,  $RB_v/N(N-1)$ . Therefore, a particular node will collapse when  $RB_v/N(N-1) > C_v$ . Considering the transferring capacity of each node is fixed to  $C$  and congestion occurs at the node with the largest betweenness centrality,  $R_c$  can be estimated as [16, 27]

$$R_c = CN(N-1)/B_{\max}, \quad (3)$$

where  $B_{\max}$  is the largest betweenness centrality of the network. This equation illuminates that the network of larger  $B_{\max}$  has smaller throughput.

For many real-life networks, the betweenness centrality is strongly correlated with degree. In general, the larger the degree, the larger the centrality. For many scale-free networks, it has been shown that the betweenness centrality approximately scales as  $B(k) \sim k^\mu$  [28, 29], where  $B(k)$  denotes the average betweenness centrality over all the  $k$ -degree nodes. Therefore, in a heterogeneous network, there exists a few high-betweenness nodes, named *hub nodes*, which are easy to be congested. This is precisely the cause of low throughput of scale-free networks.

To enhance the traffic capability, Zhao *et al.* proposed two traffic models [16], where the delivering capability of a node  $i$  is assigned as  $C_i = 1 + \beta k_i$  (Model I) and  $C_i = 1 + \beta B_i$  (Model II), respectively. Here,  $0 < \beta < 1$  is a control parameter. As we have mentioned above (see Eq. (3)), it is clear that the throughput of the whole network will increase if the hub nodes have higher delivering capability though the total capability  $\sum_i C_i$  keeps unchanged. This work suggests a way to alleviate traffic congestions for highly heterogeneous networks: making nodes with large betweenness as powerful and efficient as possible for processing and transmitting information. Particularly, in the model II, the throughput  $R_c$  is independent of the network topology.

However, there are two reasons that hinder the application of those models. First, the capability/power

distributions in some real networks are homogeneous, although their degree distributions are heterogeneous [30]. For example, in the broadcasting networks, the forwarding capacity of each node is limited. Especially, in wireless multihop ad hoc networks, each node usually has the same power thus almost the same forwarding capacity [31]. Second, the structure of real networks evolve ceaselessly, thus the degree and betweenness centrality of each node vary ever and again. By contrary, one can not change the delivering capability of a node freely due to the technical limitation.

For the case that each node has the same delivering capability  $C$ , Yan *et al.* proposed a novel routing algorithm to enhance the network throughput [32, 33]. Note that, the path with shortest length is not necessarily the quickest way, considering the presence of possible traffic congestion and waiting time along the shortest path. Obviously, nodes with larger degree are more likely to bear traffic congestion, thus a packet will by average spends more waiting time to pass through a high-degree node. All too often, bypassing those high-degree nodes, a packet may reach its destination quicker than taking the shortest path. For any path between nodes  $i$  and  $j$  as  $P(i \rightarrow j) := i \equiv x_0, x_1, \dots, x_{n-1}, x_n \equiv j$ , denote

$$L(P(i \rightarrow j) : \beta) = \sum_{i=0}^{n-1} k(x_i)^\beta, \quad (4)$$

where  $\beta$  is a tunable parameter. The *efficient path* between  $i$  and  $j$  is corresponding to the route that makes the sum  $L(P(i \rightarrow j) : \beta)$  minimum. Obviously,  $L_{min}(\beta = 0)$  recovers the traditionally shortest path length. All the information packets follow the efficient paths instead of the shortest paths.

In Fig. 2, we report the simulation results for the critical value  $R_c$  as a function of  $\beta$  on BA network with the size  $N = 1225$  and  $N = 1500$ , which demonstrate that the optimal router strategy corresponding to  $\beta = 1$  and the size of BA network doesn't affect the value of optimal  $\beta$ . In comparison with the traditional routing strategy (i.e.  $\beta = 0$ ), the throughput  $R_c$  of the whole network is greatly improved more than 10 times without any increase in algorithmic complexity. By extending the concept of betweenness centrality to efficient betweenness centrality, that is, using the efficient paths instead of the shortest paths in the definition of betweenness centrality, the analytic results can be obtained according to the Little's law [16, 27, 32]. The analytical results are also shown in Fig. 2, which agree very well with the simulations. In the previous studies, the betweenness centrality is always considered as a static topological measure of networks under the latent assumption that all the information packets go along the shortest paths from source to destination. The work of Yan *et al.* shows that this quantity (efficient betweenness) is determined both by the routing algorithm and network topology, thus one should pay more attention to the design of routing strategies. For example, a more intelligent router that can detour at obstacle performs much better than traditional

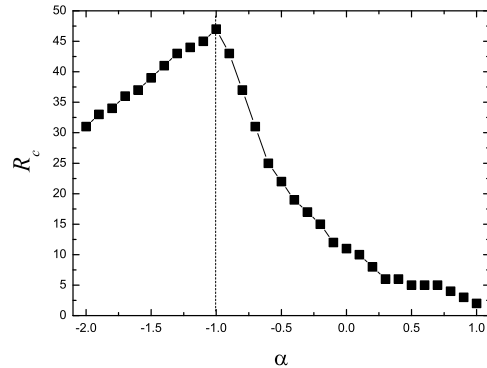


FIG. 3: The critical  $R_c$  vs  $\alpha$  for BA network with average degree  $\langle k \rangle = 10$  and of size  $N = 1000$ . The delivering capability of each node is set as a constant  $C = 10$ . The results are the average over 10 independent runs.

router which just waits at obstacle [34], and a recent work demonstrates that the dynamical information can be used to design more efficient routing strategy [35].

### III. TRAFFIC DYNAMICS BASED ON LOCAL ROUTING PROTOCOL

Although the routing protocol using global topological information is very efficient, it is not practical for huge-size communication networks and the evolving networks since the technical limitation of the router. It is because the router hardware is hard to be designed to have the capability to storage much information or adapt dynamical information. Therefore, it is also very interesting and practical to study the traffic behaviors on scale-free networks based on local information. The simplest network traffic model on local protocol is the random-walk process, where each packet is delivered to randomly selected one of its neighbors as far as it reaches the destination. The random-walk on scale-free networks has been extensively explored previously [36, 37, 38], however, it is far from the real traffic since it can not reproduce the self-similar scaling as we will present in the next section.

Motivated by the previous studies about searching engine [39, 40] and the global routing strategy [32] on complex networks, Yin *et al.* proposed a traffic model using preferential selection among local neighbors [41]. In this model, to navigate packets, each node performs a local search among its neighbors. If a packet's destination is neighboring, it will be delivered directly to its target, otherwise, it will be forwarded to a neighbor  $j$  of its currently located node  $i$  according to the preferential probability

$$\Pi_{i \rightarrow j} = \frac{k_j^\alpha}{\sum_l k_l^\alpha}, \quad (5)$$

where the sum runs over the neighbors of node  $i$ ,  $k_i$  is

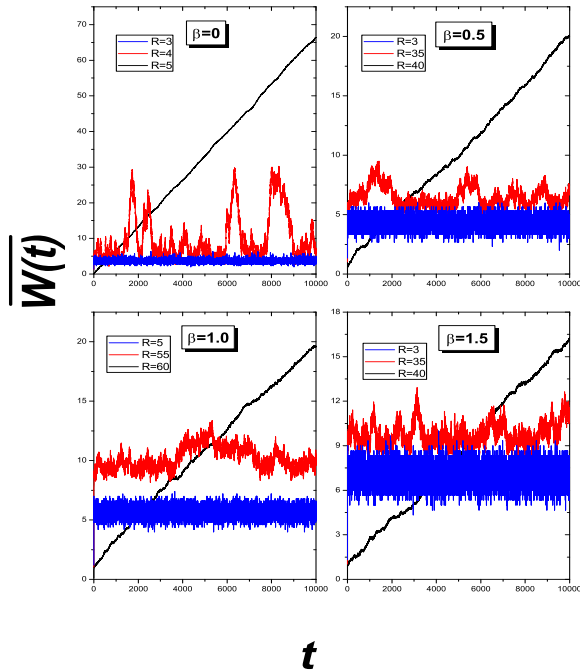


FIG. 4: (Color online) The traffic rate process for free (red), critical (blue) and congested (black) states with different  $\beta$ . All the data are obtained from an artificial traffic model [32].

the degree of node  $i$ , and  $\alpha$  is an adjustable parameter. Similar to the models mentioned in the last section, the first-in-first-out (FIFO) discipline is applied at the queue of each node. Another important rule, named path iteration avoidance (PIA), is that any edge cannot be visited more than twice by the same packet. Set the delivering capability of each node as a constant  $C$ , the simulation results show that the optimal performance of the whole system corresponds to  $\alpha = 1$  (see Fig. 3). This optimal value can also be analytically obtained [42]. Furthermore, if the delivering capability of each node is proportional to its degree, the optimal value of  $\alpha$  will shift to  $\alpha = 0$  [42].

It is worthwhile to emphasize that the behavior of Yin *et al.*'s model [41] is similar to that of Yan *et al.*'s model [32], and the behavior of Wang *et al.*'s model [42] is similar to that of Zhao *et al.*'s model [16]. These resemblances indicate the existence of some common policies between the design of global and local routing protocols, that is, to bypass the hub nodes or to improve the delivering capability of these nodes can sharply enhance the throughput of the whole network.

Note that, each router in the present models [41, 42] needs to know all its neighbors' degrees and a packet has to remember the links its has visited, which requires much intelligence of the system. It may damage the advantage of local routing strategy since to implement the PIA rule will make this system even more complicated

than the one using fixed routing algorithm. And the throughput of networks is very low without the PIA rule due to many times of unnecessary visiting along the same links by the same packets.

Another factor that affects the performance of local routing strategy is the area of information a router can make use of. Based on an artificial directed World-Wide-Web model (see some recently proposed theoretical models of directed World-Wide-Web [43, 44]), Tadić *et al.* investigated a local routing protocol with finite buffer size of each router, and found that the next-to-nearest routing algorithm can perform much better than the nearest routing algorithm [45]. In this model each packet follows a random-walk unless its destination appears within the current router's searching area, and the next-to-nearest routing algorithm means the router can directly deliver a packet to its destination if this destination is within two steps.

#### IV. THE CRITICAL PHENOMENA AND SCALING BEHAVIORS OF TRAFFIC

Recent empirical studies on communication network have found pervasive evidence of some surprising scaling properties. one example of such discoveries is that the traffic rates of both a given link in the Internet and a local Ethernet exhibit the self-similarity (or fractal-like) scaling, and the multifractal scaling is also found over small time scale [46, 47, 48, 49, 50]. These empirical studies describe pertinent statistical characteristics of temporal dynamics of measured traffic rate process and provide ample evidence that these traces are consistent with long-range correlated behavior. Furthermore, the observation of a phase transition between the free-flow phase and the congested phase in the Internet traffic is demonstrated by Takayasu *et al.* through both the round trip time experiment [51, 52] and packet flow fluctuation analysis [53, 54]. They found that the real traffic exhibits the  $1/f$ -type scaling, however, this  $1/f$  scaling can only be observed near the critical state [51, 52, 53, 54].

Cai *et al.* [55] investigated the scaling behavior of mimic traffic rate process near at the critical point generated by an the model of Yan *et al.* [32, 33]. Fig. 4 reports the average number of packets over all the nodes,  $\bar{W}(t) = W(t)/N$ , as a time series in free, critical and congested states, respectively. The behaviors of  $\bar{W}(t)$  in the free and congested states are very simple: In the former case, it fluctuates slightly around a very low value, while in the latter case, it increases linearly. However, the time series at the critical point is very complicated, which exhibits some large fluctuations like those have been observed in the real traffic [56]. This reason resulting in this phenomenon may be the usage of global routing strategy that leads to a possible long-range correlation, since this phenomenon can not be detected from the random-walk model [36] and the model based on local protocol [41, 42]. However, a very similar phenomenon is also observed in

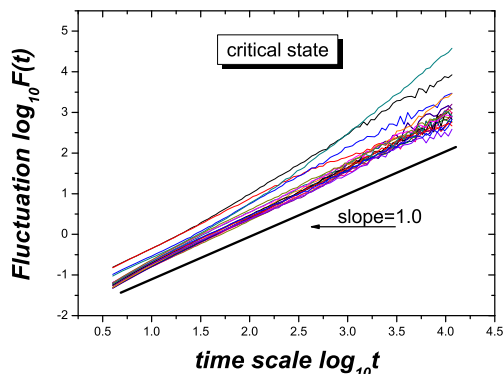


FIG. 5: (Color online) The detrended fluctuation analysis of the traffic rate processes generated by an artificial traffic model [32]. All the data are obtained from the critical state, and the different curves represent the cases of different  $\beta$  from 0 to 1.9 at step 0.1.

a traffic model with local protocol [57], where the buffer size of each router is finite.

In the previous studies, the autocorrelation function and power spectrum are widely used to analyse the self-similar scaling behaviors of both real [58, 59] and artificial [57] traffic data. However, it is shown that all the above methods do not work very well for the effect of non-stationary [60], and are less accurate than the detrended fluctuation analysis (DFA) proposed by Peng *et al.* [61, 62], which has now been accepted as an important time series analysis approach and widely used especially for financial and biological data [63, 64, 65, 66, 67].

The DFA method is based on the idea that a correlated time series can be mapped to a self-similar process by an integration. Therefore, measuring the self-similar feature can indirectly tell us the information about the correlation properties. Briefly, the description of the DFA algorithm involves the following steps.

(1) Consider a time series  $x_i, i = 1, \dots, N$ , where  $N$  is the length of this series. Determine the *profile*

$$y(i) = \sum_{k=1}^i [x_k - \langle x \rangle], i = 1, \dots, N, \quad (6)$$

where

$$\langle x \rangle = \frac{1}{N} \sum_{i=1}^N x_i. \quad (7)$$

(2) Divide profile  $y(i)$  into non-overlapping boxes of equal size  $t$ .

(3) Calculate the local trend  $y_{\text{fit}}$  in each box of size  $t$  by a least-square fit of the series, and the detrended fluctuation function is given as

$$Y(k) = y(k) - y_{\text{fit}}(k). \quad (8)$$

(4) For a given box size  $t$ , we calculate the root-mean-square fluctuation

$$F(t) = \sqrt{\frac{1}{N} \sum_1^N [Y(k)]^2}, \quad (9)$$

and repeat the above computation for different box sizes  $t$  (i.e. different scales) to provide a relationship between  $F$  and  $t$ . If the curve  $F(t)$  in a log-log plot displays a straight line, then  $F(t)$  obeys the power-law form  $t^H$  with  $H$  the slope of this line.

As shown in Fig. 5, the traffic rate time series generated by the model of Yan *et al.* [32] also exhibits the self-similar scaling behaviors at the critical point of phase transition. The scaling exponents calculated with DFA for different  $\beta$  are approximate  $H \approx 1$ , and the value of  $\beta$  has almost no effect on  $H$ . This value of  $H$  implies the  $1/f$ -type scaling in the power spectrum and the long-range correlated behavior in a wide range of scales. A very recent empirical study on the traffic rate process of a University Ethernet has demonstrated that the real Ethernet traffic displays a self-similarity behavior with scaling exponent  $\approx 0.98$  [68], which agrees well with the present result  $H \approx 1$ .

## V. CONCLUSION AND DISCUSSION

The studies of network traffic are now in the ascendant. Many new discoveries, especially the role of network heterogeneity that can sharply reduce the traffic capacity, provide us with new scenarios and problems in understanding and controlling traffic congestions. Physicists being not only the new object named *scale-free networks*, but also the new methodologies much different from those usually used in the engineering science. As an end of this brief review, we list a few interesting problems still unsolved below.

*Problem 1:* The visual field of router may be one of the most important factors that affects the traffic capacity of whole networks. In the random walk [36] the router knows nothing about the topological information; in the preferential routing strategy [41], the router knows the topological information of all its nearest neighbors; in the global routing protocol [16, 32], each router knows the whole topological information. Clearly, with wider visual field, the system can perform better. Tadić and Rodgers [45] proposed a local traffic model where each router knows the topological information of all its nearest and next-nearest neighbors, which, as expected, has obviously higher throughput than the case where only the nearest neighbors' information is available. Up to now, it lacks a systemic study on the effect of router's visual field on the traffic condition of networks, which may worth some further work.

*Problem 2:* A router can memorize huge information about shortest or efficient paths that, at least, can be used to implement the strategy of fixed routing table is

very expensive. Even worth, the current technology does not support the router to do dynamical path-finding in large-size networks. So, a relative problem to the preceding one is that what will happen if one mixes the global and local protocols together, that is, a few routers in the networks can do global path-finding or have memorized the shortest/efficient paths and others can only perform local protocol. A further question is that if the addition of a few very powerful routers to a traffic system based on local protocol can sharply enhance the network throughput, which locations should these powerful routers choose?

*Problem 3:* Although  $\beta = 1$  corresponds to the optimal value of network throughput when using efficient-path finding strategy [32], it is really a bad strategy when the traffic density is sparse since to bypass the hub nodes will increase the average distance between source and destination. If the traffic density is sparse, the strategy with  $\beta > 0$  will waste time. Therefore, a natural question raises: How to use the dynamical information to improve the performance of network traffic? Can we design some on-line algorithms to guide the information packets?

*Problem 4:* On one hand, in the network traffic dynamics, the maximal betweenness centrality  $B_{\max}$  is the key factor that determines the network throughput  $R_c$  since the node having maximal betweenness centrality is the bottleneck of information traffic thus is firstly congested. On the other hand, the node having maximal betweenness centrality is also the bottleneck that hinders the synchronization signal's communication, thus the network of higher  $B_{\max}$  may of poorer synchronizability [69, 70]. Therefore, we guess there may exist some common features between network traffic and network synchronization, although they seem completely irrelevant. Actually, some recently proposed methods used to enhance the network synchronizability can also be used to enhance the network throughput [71, 72, 73, 74]. An in-depth investigation is of great theoretical interest and we want to know if those two different kinds of dynamics, traffic and synchronization, belonging to some kind of "universality

class".

*Problem 5:* The routing strategies for real Internet [75] is of special interest for its practical significance. However, the real Internet is highly clustered and displaying hierarchical structure [5], thus far from the extensively studied BA networks. Although there exists some highly-clustered models with hierarchical structures [76, 77], they can not capture the detailed topological properties of real Internet. We have noticed that a recent model [78] aiming at Internet is very close to the reality, thus it is interesting to explore the difference between simulation results based on BA networks and the model of Zhou and Mondragón [78].

*Problem 6:* The previous studies mainly focus on the information flow and corresponding routing strategies. However, in the urban traffic, it is not the routes but the drivers are intelligent. How can they make use of traffic information to shorten their travelling time [79], and whether the selfish of each agent will reduce the system profit [80]?

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